HOW IMPROPER INTEGRALS HELP US UNDERSTAND BLACK HOLE SPACETIME

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Abstract

Although black holes may seem distant from common physics or math studies, the techniques used to analyze their peculiar properties—such as their infinitely decreasing radius —can be familiar to students. The method that is often taught in calculus for handling improper integrals can help us start an understand about black holes. Specifically, when a function's interval extends to infinity or approaches zero, we take its limit to obtain a close approximation. We will relate this to the shrinking radius of a gravitational collapse.

This paper introduces the gravitational collapse of stars and how black holes are depicted in academia. We challenge the notion of black holes as singularities, and instead choose to highlight them as infinitely curved two-dimensional planes that can be described through simple integration. By tracing the history of improper integrals, from Zeno of Elea to Pierre de Fermat, we reveal how the geometry underlying these integrals helps broaden our understanding of black holes. Our focus is on transforming an improper integral— specifically one we can link to a black hole's shrinking radius—into some form of 'limiting summation.' This is then applied to Dr. Karl Schwarzschild's radius metric derived from Einstein's field equations.

We then use this to calculate the work needed to reach 150 km above Earth's surface as its radius diminishes but its mass stays constant, increasing gravitational effects. More informally, we calculate the energy needed to reach a point above Earth as it collapses into a black hole.

We hope to show that the singularity need not pose an issue when analyzing black holes. Drawing from early geometrical philosophy, we provide an intuitive grasp of the infinite. By visually connecting concepts like unbounded intervals to objects like "Gabriel's Trumpet," we can enhance our understanding of black holes. We illustrate this with examples such as a coin

donation wells, historical arguments, and by showing how to interpret the Schwarzschild radius and the energy needed to counteract the intensified gravitational field.